Algorithm to construct Fuzzy Hyper-networks

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ABSTRACT

Generalized models of networks can be constructed as hyper-networks from reordering of the adjacency matrix of the network. The hyper-network algorithm can be formulated on the basis of a crisp representation of networks, but it is possible to extend the algorithm to fuzzy networks. Properties of the algorithm is explained in the context of point patterns in a 2D-image.

1 INTRODUCTION

Generalized models of networks can be constructed as hyper-networks from reordering of the adjacency matrix of the network [2]. Network properties identified by several researchers of the NATO IST-059 research group [1] states that fuzziness is a property of networks that should be considered. Therefore, there is a need to extend the crisp based hyper-network algorithms to handle fuzzy networks, i.e., fuzzy relations. The NATO IST-059 research group poses the question on how centrality in fuzzy networks can be computed? [1]. The present examples will indicate how the fuzzy hyper-network algorithm can be used to derive information about the centrality of groups of nodes rather than the centrality of single nodes in the original network.

2 METHOD TO DERIVE FUZZY HYPER-NETWORKS

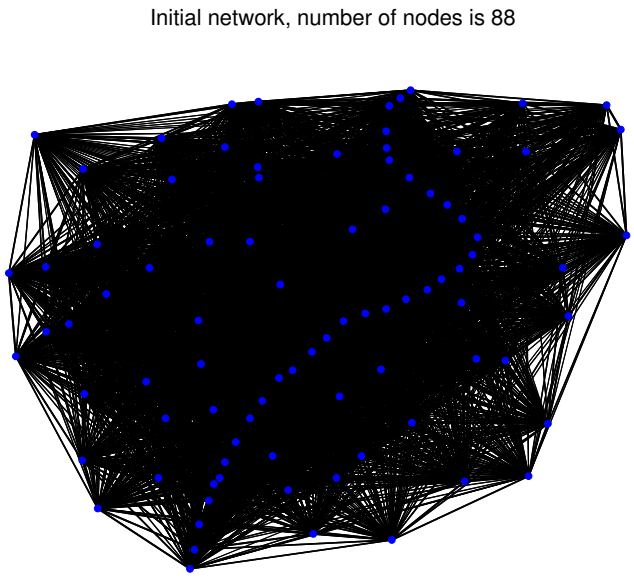
Fuzziness and uncertainty are similar concepts, but also different. For example, we can talk about high and low traffic in a network and we can talk about how certain we are about the existence of a link in a network. The latter concept is discussed by [3] and strategies to visualize uncertainty are presented. The present paper takes a different view than [3] and explores how the strength of the links in a network influences the construction of hyper-networks.

A first step in the fuzzy hyper-network algorithm is to define the numbers to be associated to the different

edges in the network. We apply the fuzzy membership function and select numbers from the closed interval zero to one, i.e., [0,1]. How to derive these numbers is an application dependant task. Then the question how to reorganize the adjacency matrix must be answered. Different methods can be selected to compute the similarity between the rows. In the forthcoming example a simple difference operator is selected. Figures 1 to 5 illustrate the method.

In Figure 1 some points in a 2D space are considered. The distance between the points is computed and membership values are associated to the different pairs of points based on the following strategy: The distance between the different nodes in the original network is mapped to a number in the interval [0,1], i.e., degree of membership. The membership value is 1 when the distance is 0 and 1 when the distance gets its maximum value in the image selected. In this way the fuzzy network is constructed.

Figure 2 and 3 show the adjacency matrix before and after the reordering. In Figure 4 the groups of nodes are computed. The applied group factor is 0.86. In Figure 5 the hyper-network after the first iteration is shown. As seen from the figure, the original image is segmented into parts of equal properties, i.e., in this case neighbourhood is measured in the embedding space, i.e., the 2D euclidian space.



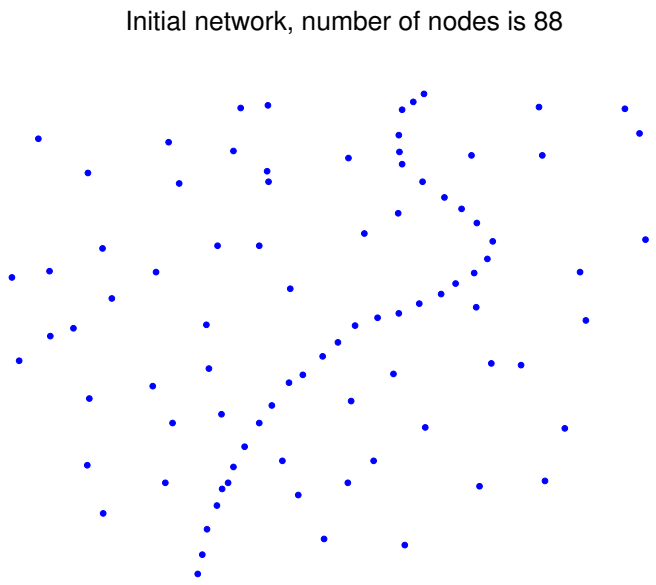


Figure 1: Point clusters in a 2D-space. A fuzzy network is generated on the basis of the nodes, by connecting all the nodes in all combinations.

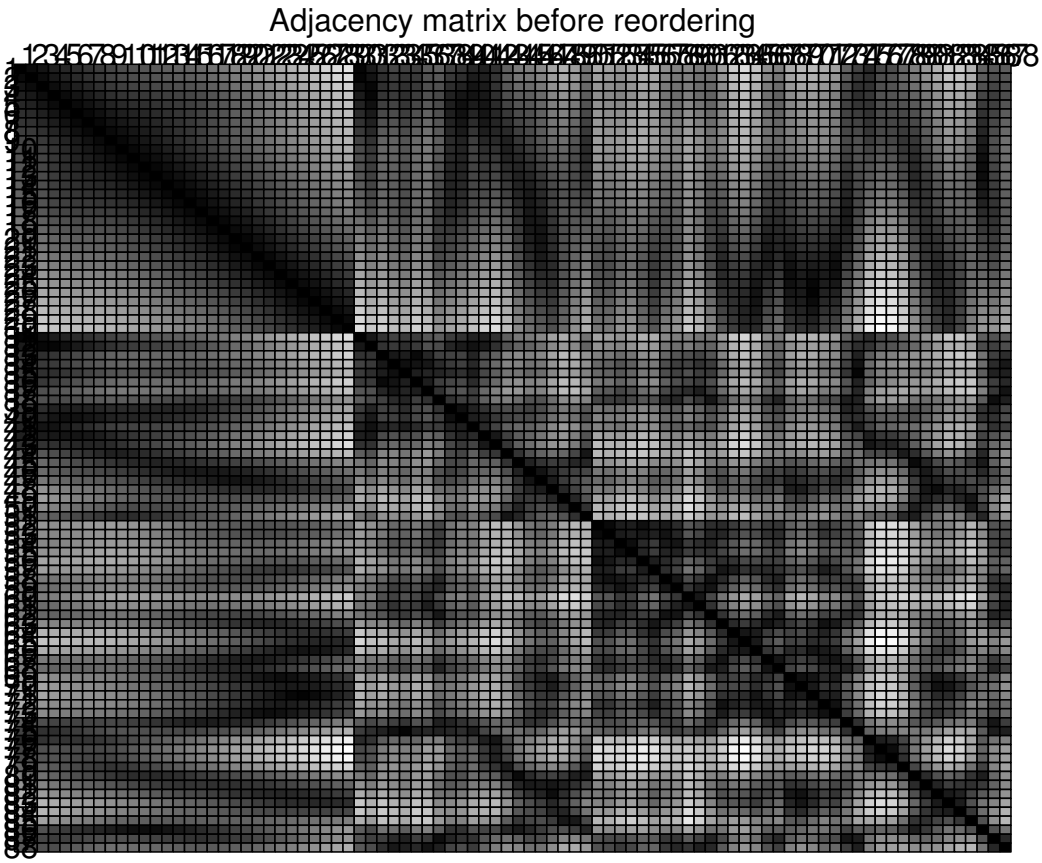


Figure 2: Adjacency matrix before the matrix is reordered

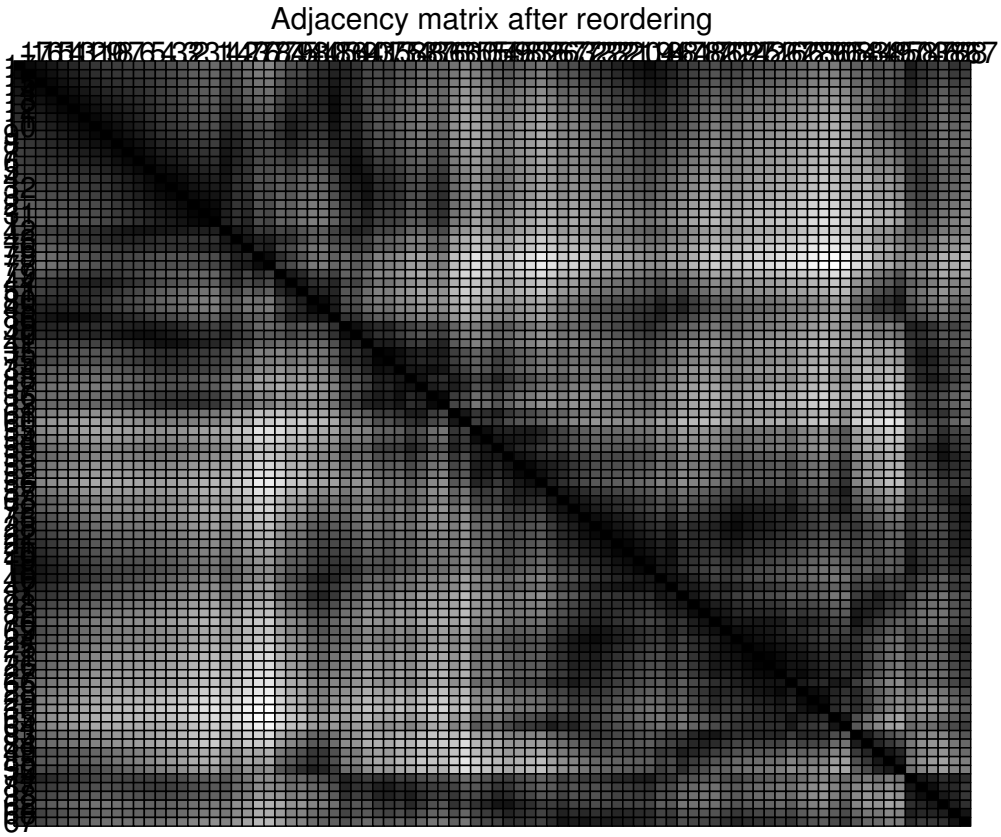


Figure 3: Adjacency matrix after the matrix is reordered. The entropy of the image is now reduced.

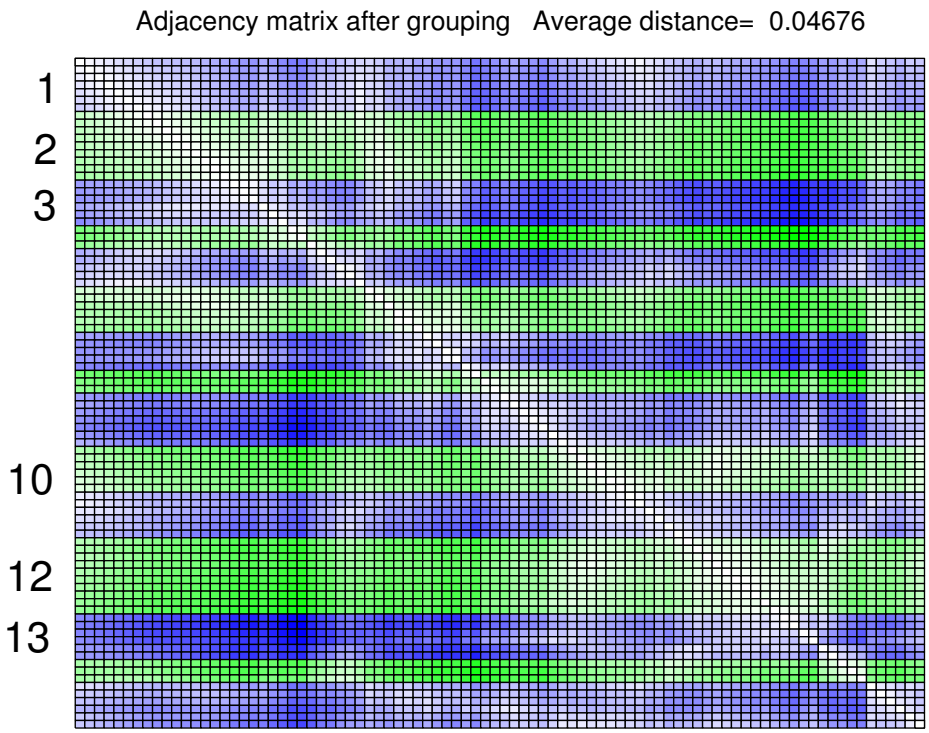


Figure 4: Groups of rows is identified, group factor 0.86. Colour hue separates the different groups. The numbers to the left identify the name of the hyper nodes shown in Figure 5.

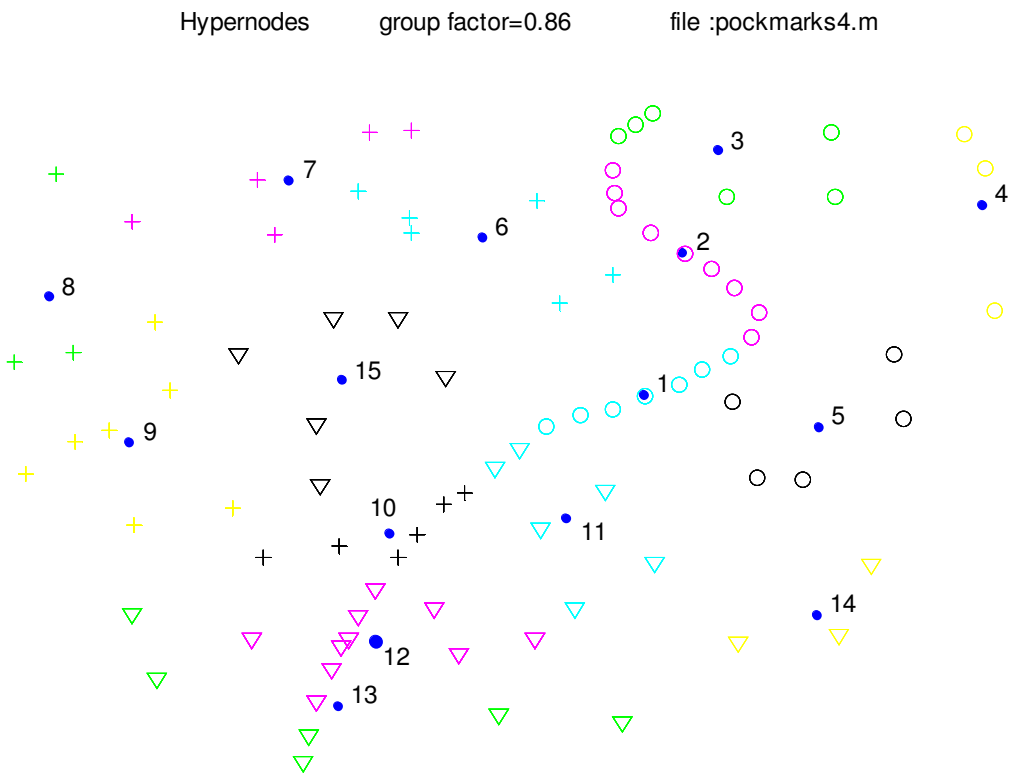


Figure 5: A fuzzy hyper-network, derived from the network in Figure 1, and its sub-nodes. In order to distinguish the sub-nodes of the different hyper-nodes, symbol colour and shape are applied. The group factor is 0.86. The distance between the different nodes in the original network is mapped to a number in the interval [0,1], i.e., degree of membership. The membership value is 1 when the distance is 0 and zero when the distance gets its maximum value in the image selected.

The next question is how to associate membership values to the hyper-links. A strategy is to apply the union operator in fuzzy set theory. The simple union operator selects the value of the strongest link. This operator can be modified by for example the Yager union operator.

CONCLUSION

Cluster analysis of points in a 2D-image is used to illustrate properties of the fuzzy hyper-network algorithm. If the algorithm is used to analyse computer networks, for example, the amount of traffic between the different nodes can be mapped to the interval [0,1]. In order to get an illustration of how the hyper-network is constructed, the nodes can be imagined embedded in a 2D-space so that the weights of the links represent the distance between the nodes. In general, there exists no metrical 2D-space that

supports the geometrical interpretation of the fuzzy network considered. Therefore, the visual imagination proposed, has limited value.

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PROVOCATION

1. Can the hyper-network algorithm add value to the understanding of the structure of fuzzy networks?
2. How can the strength of the links be mapped to the strength of the hyper links?